Mini Homework 5
Name: - Math 117 - Summer 2022

1) (2 points) Consider $V = \mathbb{C}$ as a $\mathbb{R}$ vector space. What is the complexification $V_{\mathbb{C}}$ in this case?

Solution:

2) (2 points) Let V be a real vector space. Prove that $\mathbb{R}$ linear map described below is injective

$$\begin{aligned} V &\to V_{\mathbb{C}} \\ v &\mapsto 1 \otimes v \end{aligned}$$

(This lets us "think" of V as being a subspace of $V_{\mathbb{C}}$)

Solution:

3) Let $V = \mathbb{R}[t]_{\leq 2}$, $W = M_{2\times 2}(\mathbb{R})$ with standard basis $\mathcal{B} = (1, t, t^2), \mathcal{C} = (m_1, m_2, m_3, m_4)$. Suppose $\varphi : V \to W$ is some linear map such that, under the complexified basis $\mathcal{B}_{\mathbb{C}}, \mathcal{C}_{\mathbb{C}}$ we have

$$[\varphi_{\mathbb{C}}]_{\mathcal{B}_{\mathbb{C}}}^{\mathcal{C}_{\mathbb{C}}} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 2 & 2 & 2 \\ 1 & 4 & 0 \end{pmatrix}$$

(a) (1 point) What is $\varphi_{\mathbb{C}}(1 \otimes t^2)$

(b) (2 points) What is $\varphi(1 + t)$ (not the complexified function, the original)

(c) (1 point) What is $[\varphi]_{\mathcal{B}}^{\mathcal{C}}$

Solution:

4) (2 points) Let $\varphi : \mathbb{R}^3 \to \mathbb{R}[t]_{\leq 2}$ be the linear map that sends

$$\begin{aligned} \varphi(e_1) &= t + t^2 \\ \varphi(e_2) &= 1 - t \\ \varphi(e_3) &= t \end{aligned}$$

Write the matrix of the complexified map $\varphi_{\mathbb{C}} : \mathbb{C}^3 \to \mathbb{C}[t]_{\leq 2}$ with respect to the complexified standard basis of both vector spaces.

Solution: